

Flow of two-phase media along curvilinear surfaces and in curvilinear channels is studied by simplifying the equations of motion through introduction of a special orthogonal curvilinear coordinate system.

In dealing with various technological processes and pneumohydrotransport it often becomes necessary to study two-phase flows in the working volumes of the apparatus and in curvilinear channels. The model of multivelocitly mutually penetrating continua can be used to describe the hydrodynamics of two-phase media. For example, [1-6] considered the conservation equations for the mechanics of two-phase media, while [1, 2, 5-8] dealt with the problem of forming a closed system with those equations. In particular, according to [1], when momentum transfer by pulsations can be neglected, the conservation equations can be written in the form

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} + \nabla (\rho_1 \bar{V}_1) &= 0, \\ \rho_1 \frac{d_1 \bar{V}_1}{dt} &= -\alpha_1 \nabla p + \Delta^k \bar{\tau}_1^k - \bar{f}_{12} + \bar{F}_1, \\ \frac{\partial \rho_2}{\partial t} + \nabla (\rho_2 \bar{V}_2) &= 0, \\ \rho_2 \frac{d_2 \bar{V}_2}{dt} &= -\alpha_2 \nabla p + \bar{f}_{12} + \bar{F}_2, \\ \bar{\tau}_1^{kl} &= \lambda_1^* \nabla \bar{V}_1 + 2\mu_1^* e_1^{kl}. \end{aligned} \quad (1')$$

System (1') is written with neglect of unaveraged collisions of inclusions and their shear deformations with respect to the carrier phase, the subscript 1 refers to the carrier phase, the subscript 2 to the dispersed phase, and λ_1^* , μ_1^* are viscosity coefficients.

However, solution of the mechanics equations of multiphase media is beset with great difficulties, and can be carried out only for the simplest flows, for example, one-dimensional ones. In some cases, by selection of a special curvilinear coordinate system x^i ($i = 1, 2, 3$) two-phase flows can be reduced to "close to" one-dimensional, for example, for film flows on curvilinear surfaces, flows in curvilinear axisymmetric and planar channels (tubes), etc. The coordinate system must be chosen such that the channel wall (flow surface) coincides with the coordinate surface $x^2 = \text{const}$, with the coordinate lines (surfaces) x^1 forming a family of normals to this surface. For the coordinate x^3 we choose the rotation angle for axisymmetric channels (surfaces) or the perpendicular to the channel plane for the planar case. Then the flow lines will be almost congruent to the coordinate lines x^2 , and if we then introduce the small parameter $\epsilon = l_2/l_1$, the conditions $V_{x^1} \gg V_{x^2}$ (flow close to one-dimensional) and $\partial/\partial x^2 \gg \partial/\partial x^1$ are satisfied, where l_1 and l_2 are characteristic dimensions of the flow region. In the future we shall assume that the effective viscosity of the carrier phase is sufficiently high and that the channel wall is slightly convex. Then after evaluating terms for the classes of flow enumerated above, system (1') for isothermal steady-state flows at low Reynolds numbers $Re \ll 1$ and $V_{ix^3} = 0$ takes on the form

$$\begin{aligned} -\frac{\alpha_1}{H_1} \frac{\partial p}{\partial x^1} + \frac{1}{H_1 H_2 H_3} \frac{\partial}{\partial x^2} \left[H_1 H_3 \mu (\alpha_2) \frac{H_1}{H_2} \frac{\partial}{\partial x^2} \left(\frac{V_{1x^1}}{H_1} \right) \right] \\ + \mu (\alpha_2) \frac{H_1}{H_2} \frac{\partial}{\partial x^2} \left(\frac{V_{1x^1}}{H_1} \right) - \frac{1}{H_1 H_2} \frac{\partial H_1}{\partial x^2} - f_{12x^1} + F_{1x^1} = 0, \end{aligned} \quad (1)$$

$$-\frac{\alpha_1}{H_2} \frac{\partial p}{\partial x^2} - f_{12x^2} + F_{12x^2} = 0, \quad (2)$$

$$\frac{\partial}{\partial x^1} (\alpha_1 H_2 H_3 V_{1x^1}) + \frac{\partial}{\partial x^2} (\alpha_1 H_1 H_3 V_{1x^2}) = 0, \quad (3)$$

$$-\frac{\alpha_2}{H_1} \frac{\partial p}{\partial x^1} + f_{12x^1} + F_{2x^1} = 0, \quad (4)$$

$$-\frac{\alpha_2}{H_2} \frac{\partial p}{\partial x^2} + f_{12x^2} + F_{2x^2} = 0, \quad (5)$$

$$\frac{\partial}{\partial x^1} (\alpha_2 H_2 H_3 V_{2x^1}) + \frac{\partial}{\partial x^2} (\alpha_2 H_1 H_3 V_{2x^2}) = 0, \quad (6)$$

$$\alpha_1 + \alpha_2 = 1. \quad (7)$$

In these equations the additional pressure force appearing because of fine scale perturbations is assumed equal to zero, since it is suppressed by the viscosity of the liquid phase. Then the interphase interaction force \bar{F}_{12} in the first approximation for slow motions can be represented in the form $\bar{F}_{12} = f(\alpha_2, \mu_1, \mu_2, d)(\bar{V}_1 - \bar{V}_2)$, where $f(\alpha_2) = f(\alpha_2, \mu_1, \mu_2, d)$ is the interphase interaction force coefficient. To determine the functional form of the effective viscosity $\mu(\alpha_2)$ and $f(\alpha_2) = f(\alpha_2, \mu_1, \mu_2, d)$, one can use the results of [1, 2, 5-8]. To solve system (1)-(7) we must first find the relation between the Cartesian coordinates x, y, z and the curvilinear coordinates x^1, x^2, x^3 , which is determined by the concrete geometry of the flow region, on the basis of which the Lamé coefficients H_i are defined. For example, in [9] a system of the form of Eqs. (1)-(7) was used to describe flow of two-phase mixtures in rotary mixers.

We will consider a general method for solution of system (1)-(7) for $\alpha_j = \text{const}$ ($j = 1, 2$).

1. From Eqs. (2), (5) we find the pressure

$$p = \int H_2 (F_{1x^2} + F_{2x^2}) dx^2 + c_1(x^1). \quad (8)$$

2. From Eqs. (1), (4), using Eq. (8), we obtain

$$V_{1x^1} = H_1 \left\{ \int \frac{H_2}{H_1^2 H_3 \mu(\alpha_2)} [\int c_1(x^1, x^2) H_1^2 H_2 H_3 dx^2 + c_2(x^1)] dx^2 + c_3(x^1) \right\}, \quad (9)$$

where $c_1(x^1, x^2) = \frac{c'_1(x^1)}{H_1} + \frac{1}{H_1} \frac{\partial}{\partial x^1} (\int H_2 (F_{1x^2} + F_{2x^2}) dx^2) - (F_{1x^1} + F_{2x^1})$, and $c_2(x^1)$ and $c_3(x^1)$ are determined from the corresponding boundary conditions.

3. The inclusion velocity V_{2x^1} is determined from Eq. (4) with consideration of Eqs. (8), (9).

4. Then, Eqs. (3), (2), (5) are used to find the unknowns V_{1x^2} and V_{2x^2} , while in the general case Eq. (6) can be used to determine α_2 .

5. To find the unknown $c'_1(x^1)$ the condition of constancy of mass flow rates

$$\sum_{i=1}^2 \int_{x_i^2}^{x_f^2} \int_{x_i^3}^{x_f^3} \alpha_i \rho_i^0 V_{ix^1} H_2 H_3 dx^2 dx^3 = q_1 + q_2. \quad (10)$$

may be used.

The simplified system of equations (1)-(7) and this solution technique have been used to calculate the motion of two-phase flows in a conical tube and single-cavity hyperboloid of revolution, and to describe the hydrodynamics of centrifugal separators used to separate suspensions and rotary mixers used for preparation of composition materials containing a solid phase [9].

When the above solution technique is applied to the case of a two-phase system in the cavity of a single-cavity hyperboloid of revolution, the orthogonal curvilinear coordinate system chosen is that of an oblate ellipsoid of revolution x^1, x^2, x^3 , related to the cylindrical coordinate system z, r, φ by the expressions

$$z^2 = -r_0^2 x^1 x^2, \quad r^2 = r_0^2 (1 + x^1)(1 + x^2), \quad \varphi = x^3 \quad (11)$$

over the range $0 \leq x^1 \leq \infty, -1 \leq x^2 \leq 0, 0 \leq x^3 \leq 2\pi$. The Lamé coefficients then have the form [10, 11]

$$H_1 = \frac{r_\alpha}{2 \sin \alpha} \sqrt{\frac{x^1 - x^2}{x^1(1 + x^1)}}, \quad H_2 = \frac{r_\alpha}{2 \sin \alpha} \sqrt{\frac{x^1 - x^2}{-x^2(1 + x^2)}},$$

$$H_3 = \frac{r_\alpha}{\sin \alpha} \sqrt{(1 + x^1)(1 + x^2)}, \quad (12)$$

where 2α is the apex angle, r_α is the radius of the throat of the hyperboloid, and r_0 is one-half the focal length. In the case of a hyperboloid of revolution oriented vertically in a gravitational field, the projections of the mass forces \bar{F}_i on the axes x_1 and x_2 have the form

$$F_{ix^1} = -\sqrt{\frac{-x^2(1 + x^1)}{x^1 - x^2}} \rho_i g, \quad F_{ix^2} = \sqrt{\frac{x^1(1 + x^2)}{x^1 - x^2}} \rho_i g. \quad (13)$$

Then the solution of Eqs. (1)-(7) can be represented in the form

$$V_{1x^1} = H_1 c_1'(x^1) F_0(x^1, x^2) / \mu(\alpha_2), \quad (14)$$

$$V_{1x^1} - V_{2x^1} = (\alpha_2 F_{1x^1} - \alpha_1 F_{2x^1} + \alpha_2 c_1'(x^1) / H_1) / \bar{f}(\alpha_2), \quad (15)$$

$$V_{1x^2} - V_{2x^2} = (\alpha_2 F_{1x^2} - \alpha_1 F_{2x^2}) / \bar{f}(\alpha_2), \quad (16)$$

$$\begin{cases} p(x^1) - p(x_{in}^1) = \int_{x_{in}^1}^{x^1} \left(\frac{\partial p}{\partial x^1} \right)_{av} dx^1, \\ \frac{\partial p}{\partial x^1} = c_1'(x^1) + H_1 (F_{1x^1} + F_{2x^1}), \end{cases} \quad (17)$$

$$c_1'(x^1) = ((q_1 + q_2) / 2\pi + \Phi_1(x^1, x_f^2) / \Phi_2(x^1, x_f^2)), \quad (18)$$

where

$$F_0(x^1, x^2) = \Phi_0(x^1, x_f^2) - \Phi_0(x^1, x^2); \quad \Phi_0(x^1, x^2) = \frac{4}{3} x^1 (1 + 3x^1)$$

$$\times \left(\frac{1}{\sqrt{x^1}} \operatorname{arctg} \sqrt{\frac{-x^2}{x^1}} + \ln(1 + \sqrt{-x^2}) - \frac{x^1}{1 + 3x^1} \ln(x^1 - x^2) \right);$$

$$\Phi_1(x^1, x^2) = \frac{\alpha_1 \alpha_2^2 \rho_2^0 (\rho_2^0 - \rho_1^0) g}{2 \bar{f}(\alpha_2)} \left(\frac{r_\alpha}{\sin \alpha} \right)^2 (1 + x^1)(1 + x_f^2);$$

$$\Phi_2(x^1, x_f^2) = \frac{2 \alpha_2^2 \rho_2^0}{\bar{f}(\alpha_2)} \left(\frac{r_\alpha}{\sin \alpha} \right) (1 + x^1) (1 - \sqrt{-x_f^2}) \sqrt{x^1} -$$

$$- \frac{\alpha_1 \rho_1^0 + \alpha_2 \rho_2^0}{9 \mu(\alpha_2)} \left(\frac{r_\alpha}{\sin \alpha} \right)^3 \sqrt{x^1} (1 + 3x^1) \left(\frac{2(1 + x^1)}{3(1 + 3x^1)} (1 - (-x_f^2)^{\frac{3}{2}}) + \right.$$

$$\left. + (1 - \sqrt{-x_f^2}) \left(\frac{10(x^1)^2 + 12x^1 + 2}{1 + 3x^1} \right) + \left(\operatorname{arctg} \frac{1}{\sqrt{x^1}} - \right. \right.$$

$$\left. \left. - \operatorname{arctg} \sqrt{\frac{-x_f^2}{x^1}} \right) \left(\frac{8(x^1)^{\frac{5}{2}}}{1 + 3x^1} - \frac{2(1 + 3x^1)}{\sqrt{x^1}} \right) + \right.$$

$$\left. + (-4x^1) \ln \left(\frac{x^1 - x_f^2}{1 + x^1} \right) + 4(1 + 3x^1) \ln \left(\frac{1 + \sqrt{-x_f^2}}{2} \right) \right),$$

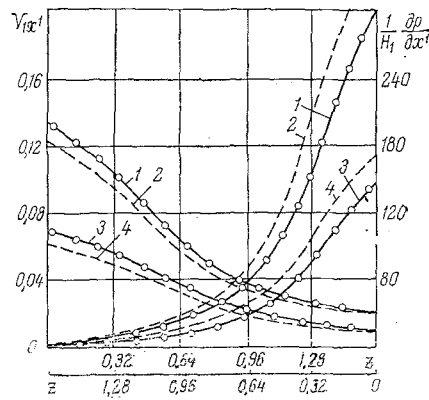


Fig. 1. Longitudinal velocity V_{1x^1} and pressure along tube axis $z = r_0 \sqrt{x^1}$ and comparison at $\alpha_2 = 0$ with solution of [11] for various flow parameter values: 1) $q_1 = 0.63$ kg/sec, $g = 0$ m/sec², $\alpha_2 = 0$; 2) $q_1 + q_2 = 0.63$ kg/sec, $g = 9.81$ m/sec², $\alpha_2 = 0.1$; 3) $q_1 = 0.315$ kg/sec, $g = 0$ m/sec², $\alpha_2 = 0$; 4) $q_1 + q_2 = 0.315$ kg/sec, $g = 9.81$ m/sec², $\alpha_2 = 0.1$; lower abscissa indicates pressure head, $\rho_1^0 = 1260$ kg/m³, $\rho_2^0 = 2520$ kg/m³, $\mu_1 = 1.4$ N·sec/m². Points, calculation of Eq. (11) with corresponding parameter values; solid lines, single-phase flow; dashed lines, flow of carrier phase of two-phase liquid with identical flow parameters. z , m; V_{1x^1} , m/sec; $(1/H_1) \cdot dp/dx^1$, N/m³.

$(\partial p / \partial x^1)_{av}$ is the mean (over channel section) pressure head.

Given specified phase mass flow rates, Eqs. (13)-(18) may be used to calculate all flow parameters, while with unknown mass flow rates Eqs. (17)-(18) can be used to determine $q_1 + q_2$ for a specified pressure head.

A comparison of the velocity V_{1x^1} and the pressure change along the tube axis at $\alpha_2 = 0$ with the solution of the analogous problem for a single phase Newtonian medium [11] showed good agreement (Fig. 1).

The solution of Eqs. (1)-(7) for a conical tube in a spherical coordinate system r, θ, φ has the form

$$V_{1r} = \frac{r^2 c_1'(r)}{\mu(\alpha_2)} \ln \left(\frac{1 + \cos \theta_0}{1 + \cos \theta} \right), \quad (19)$$

$$V_{1r} - V_{2r} = (\alpha_1 \alpha_2 (\rho_2^0 - \rho_1^0) g \cos \theta + \alpha_2 c_1'(r)) / f(\alpha_2), \quad (20)$$

$$V_{1\theta} - V_{2\theta} = (\alpha_1 \alpha_2 (\rho_1^0 - \rho_2^0) g \sin \theta) / f(\alpha_2), \quad (21)$$

$$c_1'(r) = \frac{(q_1 + q_2) / 2\pi + \alpha_1 \alpha_2^2 \rho_2^0 (\rho_2^0 - \rho_1^0) g r^2 \sin^2 \theta_0 / 2f(\alpha_2)}{(\rho_1 + \rho_2) r^4 \left(2 \ln \left(\frac{1 + \cos \theta_0}{2} \right) + 1 - \cos \theta_0 \right) / \mu(\alpha_2) + \alpha_2 \rho_2 r^2 (\cos \theta_0 - 1) / f(\alpha_2)}, \quad (22)$$

where θ_0 is the apex angle.

If we assume that the pressure change is constant along the tube length ($c_1'(r) = 0$) and $\bar{V} = \alpha_1 \bar{V}_1 + \alpha_2 \bar{V}_2 = 0$, then from Eqs. (20), (21) we obtain the spherical particle precipitation rate in a gravitational field presented in [5]:

$$\bar{V}_2 = (1 - \alpha_2)^2 \left(1 - \frac{5}{2} \alpha_2 \right) \frac{2(\rho_2^0 - \rho_1^0) d^2 - g}{9\mu_1} \quad (23)$$

$$\text{at } f(\alpha_2) = \frac{9\alpha_2\mu_1}{2d^2 \left(1 - \frac{5}{2} \alpha_2 \right)}$$

At $\alpha_2 = 0$ Eqs. (19)-(22) agree well with known solutions for the flow of a single-phase liquid in a conical tube [12], with the relative error for the longitudinal velocity not exceeding 5% at $\theta_0 \leq 30^\circ$. The solution obtained for the conical tube as $\theta_0 \rightarrow 0$ describes flows of two-phase media in cylindrical tubes.

The simplification of Eqs. (1)-(7) and this solution technique can also be used for study of the flow of one- and two-phase media in other curvilinear channels and tubes. For example, Eqs. (1)-(10) describe the flow of two-phase mixtures in planar curvilinear channels at $H_1 = H_1(x^1, x^2)$, $H_2 = H_2(x^1, x^2)$, $H_3 = 1$, while at $H_1 = 1$, $H_2 = H_2(x^2, x^3)$, $H_3 = H_3(x^2, x^3)$, they describe flow in cylindrical tubes of noncircular cross section if we can introduce the small parameter $\varepsilon = l_2/l_3$, where l_2 and l_3 are the characteristic tube dimensions in the directions x^2 and x^3 . In this case the orthogonal curvilinear coordinates x^2 and x^3 are related to the configuration of the tube section, while $x^1 \equiv z$ and is directed along the tube axis. The expression for the longitudinal velocity has the form

$$V_{1x^1} = - \int_{x_1^2}^{x_2^2} \frac{H_2}{H_3\mu(\alpha_2)} \left(\int_{x_1^3}^{x_2^3} (\partial p / \partial z - F_{1x^1} - F_{2x^1}) dx^2 + c_2(x^3) \right) dx^2.$$

In conclusion, it should be noted that in solution of the problem we have assumed that the inclusion concentration is constant over the entire flow region and that adhesion conditions are satisfied on the tube walls. But these assumptions are only approximations to the real conditions. In reality, the concentration can be considered constant only outside a wall layer with thickness on the order of the particle size under concrete flow conditions, while the adhesion boundary conditions must be replaced by other ones. The effect of external boundaries on flow of two-phase media has been considered in detail in [2, 8].

NOTATION

\bar{V}_j , ρ_j , α_j , velocity, mean density, and volume concentration of phase j ; ρ_j^0 , true density of phase j ; $\bar{\tau}_i^{k\bar{l}}$ and $e_i^{k\bar{l}}$, stress and deformation rate tensors; \bar{f}_{12} , interphase interaction force; \bar{F}_j , mass force acting on phase j ; p , pressure; $f(\alpha_2)$ and $\mu(\alpha_2)$, interphase interaction and effective viscosity coefficients of mixture; x^i , orthogonal curvilinear coordinates; H_i , Lamé coefficients; q_j , mass flow rate of phase j ; d , characteristic dimension of inclusions; μ_1 , viscosity of liquid phase 1; x_1^i and x_2^i , initial and final x^i values.

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MAGNETOGASDYNAMIC MODEL OF CAPILLARY DISCHARGE
FROM EVAPORATING WALL

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The article describes the mathematical and physical models of heavy-current capillary discharges. The results of numerical calculation of plasma flow in capillaries are presented.

Capillary discharge from an evaporating wall (CDEW) is widely used in standard light sources ÉV-45 [1] and "Impul's-5" [2] as source of plasma with controlled parameters. This makes it possible to investigate the thermodynamic and optical characteristics of the plasma, and also processes occurring in plasma jets, etc. The experimental study of CDEW is limited on account of its specific features: a relatively cool, optically dense plasma shell adjacent to the walls of the capillary prevents us from obtaining direct information on the parameters prevailing in the hot region of the discharge near the axis. Yet this region may play a decisive part in the overall energy balance and mass balance of the evaporated substance, especially when radiant transfer is the dominant process of energy supply to the wall [3]. Another equally important circumstance stimulating interest in the theoretical investigation of CDEW is the strong nonideality of plasma in capillaries that leads to plasma phase transformation [3]. Also of interest is the study of plasma with higher parameters than in the gasdynamic regime of CDEW, induced and maintained by currents of heavy-current pulse discharge in the gas, the intensity being $\sim 10^5$ A or more [4, 5].

Thus, working out a theoretical model of CDEW and preparing on its basis a program of calculating the dynamics of the phenomenon makes it possible to reveal processes and parameters that are inaccessible to direct experimental observations. This, in turn, makes it possible to influence in a controlled manner the quantitative characteristics of CDEW.

Below we describe the physical and mathematical models of heavy-current capillary discharges when the magnetic eigenfield of the discharge current is of high intensity, and the magnetic pressure is comparable with the gas-kinetic pressure. We also present the results of numerical calculations of radiative and gasdynamic processes occurring in capillary discharges from the initial nonsteady-state phase up to the establishment of steady-state plasma motion.

To describe plasma flow in the channel of a capillary discharge with heavy discharge currents, when magnetic pressure may not be neglected in comparison with the gas-kinetic pressure, we use a system of magnetic and gasdynamic equations supplemented by Maxwell's equations of the electromagnetic field [6]. To construct the model of the flow, we make some estimates. The test of freezing of the magnetic field in the plasma is the magnetic Reynolds number Re_H . The characteristic parameters for the magnetic and gasdynamic (MGD) flow regime of plasma in

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